The GHZ Version of Bell's Argument

Malcolm Forster, 14 April, 1997

The conclusion of Bell's argument is that there is no (plausible) causal explanation of some experimentally observed correlations. It is a *reductio ad absurdum* argument – it shows that the only (plausible) causal explanations lead to an absurdity (a false prediction), and are therefore false.

Three particles (electrons) fly apart towards 3 widely separated measuring devices, A, B, and C, each of which can be put at one of 2 settings, labeled 1 and 2. Each measurement device has a light bulb attached, which flashes red or green when a particle enters it. Which color the bulb flashes is not controlled by the experimenter. This is the outcome of the experiment. The experimenter does control the *settings* at position 1 or position 2 (in the actual experiment, the setting determines the orientation of a magnet). Note that the settings are quite independent of the outcome—any device can flash red or green in either setting.

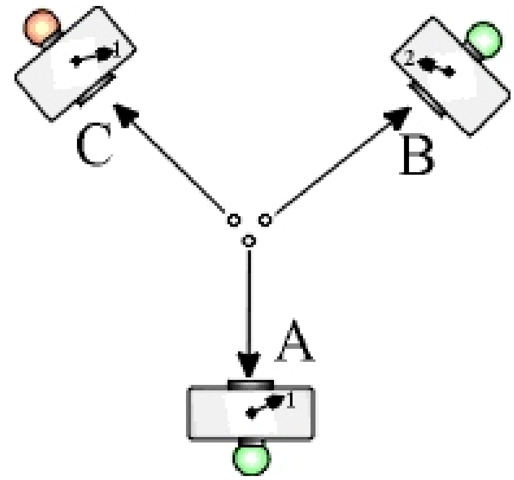

Figure: The setting is 1–2–1 and the outcome is G–G–R.

FIRST EXPERIMENTAL FACT: When any 2 of the 3 measurement devices are at setting 2 and the other is at setting 1, then there is an odd number of red lights flashing in every instance–that is, either all 3 bulbs flash red, or exactly one bulb flashes red (with 2 green flashes) in any order. Each of these four possibilities occurs with equal frequency. A quick way of summarizing this fact is that there is always an odd number of red flashes in any of the settings 1–2–2, 2–1–2, or 2–2–1. Here are the kind of predictions we can make using this experimental regularity in these kinds of experimental setup: If two devices flash red, then the other must also be red. And if one is red and the other green, then the third must be green.

The only PLAUSIBLE causal explanation of such correlations is this: Suppose that each particle, after separation from the others, carries with it a set of "instructions." That is, each particle carries instructions that DETERMINE which color will flash for each of the two possible settings of the device it enters. Let us represent by $A1 = +1$ the instruction that the particle approaching the A device cause the red light to flash if the device is set to position 1. And the instruction that the green light flash if the device A set to position 1 by $A1 = -1$. Similarly, let $A2 = +1$ and $A2 = -1$ determine the outcome when the measuring device is set at position 2. Since at most one of the variables A1 and A2 will be measured, they are commonly called *hidden variables*. So, the particle heading towards A will have exactly one of 4 possible sets of instructions: Either:

$\{A1 = +1, A2 = +1\}$, $\{A1 = +1, A2 = -1\}$,
$\{A1 = -1, A2 = +1\}$, or $\{A1 = -1, A2 = -1\}$.

Note that in a single run of the experiment, the measurement device cannot be set at both settings at once, so we could not know what its full instruction set was (even if it had one). For instance, if we see the red flash when the device is set to 2, then we would only know that the instruction set was either $\{A1 = +1, A2 = +1\}$ or $\{A1 = -1, A2 = +1\}$. This is why the variables A1, A2, etc, are called HIDDEN variables, for they are not known before the experiment begins and they are not all known even after the experiment ends. Note that each particle is destroyed when it is measured so we cannot run the same particle through the measurement device on the other setting. And even if we could, we could never be sure that the instruction set had not been DISTURBED by the first measurement.

On this hidden variable story, each of the 3 particles is supposed to carry it own set of instructions (which are allowed to arranged in *any* way you want). But, in order to account for the experimental facts described so far, we must assume that the particles mutually co-ordinate their instructions before they separate so as to produce the right correlations. This is the usual way in which causal stories explain correlations. For example, suppose that we that observe that two widely separated television sets always show the same picture, so that knowing what picture appears on one enables us to predict the what picture is appearing on the other with certainty. How do we explain this regularity, or correlation? The correct causal explanation of this correlation is that each TV is tuned to the same television station, and is thereby responding to a COMMON causal influence. This is referred to as a "common cause explanation." We certainly do not need to suppose that there is any DIRECT causal link between the correlated events. Similarly, it is a common cause explanation that is being tried out as an explanation of the quantum phenomena described. So, to give a COMMON CAUSE ACCOUNT of the experimental facts described so far, we need to suppose is that the 3 particles "conspire" before separation in such a way that the possible instruction sets they leave with satisfy the following 3 constraints:

$$A1 \times B2 \times C2 = +1, \ \ A2 \times B1 \times C2 = +1 \ ,$$
$$\text{and } A2 \times B2 \times C1 = +1 \ .$$

The first equation ensures that there is an odd number of red flashes when the setting is 1–2–2. The second equation ensures the same thing when the setting is 2–1–2, and the third when the setting is 2–2–1. These equations are part of the EXPLANANS–they are not merely "statements of experimental fact." They *explain* the first experiment fact.

So far, so good. BUT, we may make a PREDICTION from this causal explanation. For it is a simple mathematical fact that: $(A1 \times B2 \times C2) \times (A2 \times B1 \times C2) \times (A2 \times B2 \times C1) = A1 \times B1 \times C1 \times (A2 \times A2) \times (B2 \times B2) \times (C2 \times C2) = A1 \times B1 \times C1$ (since $A2 \times A2$ is always +1 whether or not A2 is +1 or –1. Similarly, $B2 \times B2 = 1$ and $A2 \times A2 = 1$). It now follows from our three equations that $A1 \times B1 \times C1 = 1$, and this leads to the PREDICTION that an ODD number of red flashes will occur when the setting is 1–1–1.

This prediction makes some assumptions. It assumes that the constraint that ensures that the first experimental regularity must hold remains in place when a new kind of setting is in place (namely 1–1–1). This is a very plausible assumption, since to assume otherwise would be to assume that the instruction sets are fixed with advanced 'knowledge' of the future experimental setting.

With this assumption we have a prediction that goes beyond the first experimental fact. And as Hume pointed out, it is logically possible that such predictions are wrong. In this case, the prediction is wrong.

SECOND EXPERIMENTAL FACT: There is always an EVEN number (0 or 2) of red flashes when all 3 devices are set to position 1. So, the prediction is *false!*

THEREFORE, the preceding hidden variable story is false.

There are at least (in principle) two ways of salvaging some sort of causal explanation of these strange quantum mechanical correlations–but neither alternative is very attractive.

1. Postulate some DIRECT causal influence that propagates from one measurement event to another so that the instruction sets are changed "in flight." But the experiment can easily be set up in such a way that all 3 measurement events occur simultaneously (relative to some frame of reference) at widely separated positions. In this case the postulated causal influence would have to travel at a speed faster than the speed of light, which conflicts with one of the basic postulates of Einstein's theory of relativity. So, this "non–local" causal story is not an attractive alternative.
2. Suppose that the particles "know" what the measurement settings will be so that the equation that they violate is not one that will affect the experimental results in the setting they will actually face. Then the RELEVANT equation is always be satisfied, thereby producing the correct *experimental* correlations. However, such a RAPPORT between instruction sets and measurement settings is unconfirmed at least and highly implausible at best.

REFERENCE:

N. David Mermin, "Quantum Mysteries Revisited." *American Journal of Physics*, August, 1990, pp.731–734.

REMARK 1: While quantum mechanics does predict with certainty that an odd number of red flashes occur when in the 1–2–2, 2–1–2, and 2–2–1 settings and why an even number of flashes occur when the setting is 1–1–1, the theory does not predict whether a particular light flashes red or green. The probability of each alternative is ½. We can see this by considering the case in which an odd number of red flashes occurs (the other case is the same). For quantum mechanics predicts that the 4 possible outcomes R–R–R, R–G–G, G–R–G, and G–G–R occur with equal frequently in the long run (each with probability 1/4-like tossing a 4-sided die). Moreover, without hidden variables, this is the best we can ever do. So, in addition to showing that not all explanation is causal, Bell's argument also shows that science should *limit* its demand for explanation (causal or otherwise). Another example is the law of inertia, according to which we do not need to postulate forces to explain uniform rectilinear motion.

REMARK 2: Bell's argument undermines all CAUSAL views of explanation *including probabilistic causal accounts*, since one still needs deterministic causes to causally explain strict corre-