

# Prediction and the Problem of Induction

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## 1 The Problem of Knowledge

Epistemology is commonly defined as the study of the origins and nature of *knowledge*, where knowledge is true, justified belief. This is a strong characterization of the subject matter of epistemology. It is so strong that skeptics have mounted good arguments to the conclusion that we know very little, if anything at all. Even if the skeptics are right, there remains the fact that many false, and unjustified, beliefs provide reliable predictions that are approximately true. Meteorology, nowadays, is remarkably accurate although forecasts are not often *exactly* true. The daytime high is forecast to be 15°C, but it reaches 16°C. Should epistemologists ignore meteorology as an interesting object of study? I think not. This book is about the epistemology of science in a broader sense of epistemology than commonly conceived in modern philosophical circles.

In search of a broader conception of the problem, let me remind you of Hume's problem of induction. Suppose we consider our belief that "All billiard balls will move when struck at any time." Although this belief is not true (billiard balls glued to the table will not move when struck), it does yield reliable predictions in ordinary circumstances. The epistemological problem here is: What are the intelligent processes by which such predictions are made and what explains their reliability (to the extent that they are reliable)? Epistemologists have always sought a *general* answer to this question. It's no good spinning a different yarn for each case. What's wanted is some *general explanation* that would immediately cover any other kind of example. It is not tied to the peculiarities of bil-

liard balls, or the particular characteristics of the predictor. It has to be subject-neutral. Classically, logic is the tool supposed to provide such subject-neutral answers to such epistemological questions.

The method of *enumerative induction* is the simplest and most naive answer to the question about how reliable predictions are made: If past experience has yielded a constant conjunction of cases in which objects with property *A* also have property *B*, with no exceptions, then we draw the conclusion that all *As* are *Bs* and predict that the next *A* will be a *B*. Notice that the answer is formulated in a subject-neutral way. In our example, *A* is the property of being a struck billiard ball, and *B* is the property of being of moving. But the answer also covers other examples, and this is an attractive feature.

Hume's critique of induction shows that the method of enumerative induction does not yield justified true belief, where 'justified' means 'justified with certainty'. Consider any inference of the form:

Billiard ball 1 moved when struck at time 1.

Billiard ball 2 moves when struck at time 2.

⋮

Billiard ball 100 moves when struck at time 100.

*Therefore*, Billiard ball 101 will move when struck at time 101.

Since it is logically *possible* for the prediction to be false while past evidence is true, the evidence does not conclusively establish the truth of the prediction. Hume's point is that no *infallible* justification of the prediction is possible, and Hume's *problem of induction* is the problem of

*justifying* the trust we place in any prediction.

One kind response to Hume's argument has been to deny that enumerative induction is the correct method of prediction in science. Hume's argument still holds no matter what method produces the prediction. Maybe the prediction is made from some hypothesis that has survived the severest attempts at falsification. Such a method is not inductive, as Karl Popper (1959) has long emphasized, but it does not affect Hume's conclusion: No method of producing predictions from past evidence will guarantee their truth.

Another kind of response has been to deny that the *truth* of our predictions is what we need to justify. Maybe we only need to justify their approximate truth, or their probability of truth? However, Hume's point is not restricted to conclusions about the *truth* of predictions. It is also applies to any claim that "transcends" experience. There is no *infallible* way of justifying claims about the *probability* of our predictions, or about the approximate reliability of predictions, or about their degree of accuracy.

A third, and final, response is that our predictions are justified because we know that the future is like the past in some respects. We have to know the respect in which nature is uniform, and it must be relevant to the inference. The objection to this response is that there is no infallible justification for the assumption of uniformity. Either the uniformity principle is based on other past experience, in which case the problem of induction arises "one step back," or it is not justified. In either case, an appeal to the uniformity of nature does not help.

In my view, the only correct complaint about Hume's argument is that it is irrelevant to the proper subject matter of epistemology. Hume is right that none of our predictive beliefs are *justified* in the strong sense that he demands, but that does not mean that epistemology is an empty discipline, or a branch of psychology (as Hume

thought). On the positive side, I propose that two key questions are central to epistemology; one is a how-question, and the other is a why-question.

- *How* are *predictions* made from past observations?

Once the how-question is correctly answered, we may also ask:

- *Why* are predictions (made in this way) true, or approximately true?

This why-question does not presuppose that any method of prediction can be justified. An *explanation* of why enumerative induction works as well as it does might appeal to the uniformity of nature, and it is no objection to point out that such an assumption cannot be justified. Explanations commonly appeal to assumptions that cannot be justified; in fact Hume's argument shows that they invariably appeal to such assumptions. Yet explanations are successfully provided within many disciplines, and there is no reason why epistemology should be an exception.

Perhaps, the traditional replies to Hume's problem will prove useful to the revised project after all? However, that is doubtful because enumerative induction is not the right answer to the how question. To ask why enumerative induction works is to presuppose that it is the method that science has used. There is every reason to believe that the presupposition is false. The remainder of this chapter is devoted to finding a better, more complete, answer to the how-question. In the next section, I will start with something that is fairly close to simple enumerative induction, and then modify the proposal in later sections.

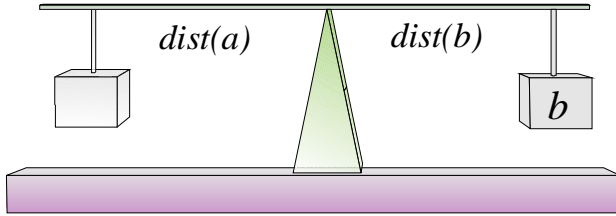


Figure 1: The balance beam

## 2 A Scientific Example

Suppose we hang an object, called *a*, on one side of a beam balance, and find how far a second object *b* has to hang on the other side to balance the first. The distance that *a* is hung from the point at which the beam is supported (called the *fulcrum*) is labeled *dist(a)*, while the distance to the right of the fulcrum at which *b* balances *a* is labeled *dist(b)*. If *b* is moved to the left of this point then the beam tips until the object *a* rests on the ground and if *b* is moved to the right the beam tips the other way. In the first instance, *dist(a)* is 1 centimeter (cm) and the beam balances when *dist(b)* is 2 cm. This pair of numbers is a *datum*. We repeat this procedure 5 more times with different values of *dist(a)*, and tabulate the resulting data in Table 1. We are now asked to make the following *prediction*: Given that *a* is hung at a distance of 2 cm to the left of the fulcrum, *predict* the distance at which *b* will balance; *viz.* predict *dist(b)*. Once we notice that for all six data, *dist(b)* is twice *dist(a)*, it appears

	<i>dist(a)</i>	<i>dist(b)</i>
datum 1	1 cm	2 cm
datum 2	4 cm	8 cm
datum 3	3½ cm	7 cm
datum 4	2½ cm	5 cm
datum 5	1 cm	2 cm
datum 6	5 cm	10 cm

obvious that we should predict that *b* must be 4 cm when *a* is hung at 2 cm.

The naive answer to the question about *how* to make predictions is roughly: Regularities are observed in past data, and these regularities are then applied to future instances. In the beam-balance example, this idea may be spelt out in terms of an argument: In all the *observed* instances, object *b* balanced at twice the distance as object *a*. *Therefore*, *b* will balance at twice the distance of *a* in the *next instance*. The general pattern, or *form*, of the argument for the prediction will be called “*Next-Instance*” *Induction*:

In all the *observed* instances, system *s* conforms  
to a regularity

System *s* will conform to regularity *R* in the next  
instance.

In our example, the *system s* is the beam balance with objects *a* and *b* hung at opposite sides of the fulcrum, and the regularity *R* may be expressed by the mathematical equation:

$$R: \quad \textit{dist}(b) = 2 \textit{dist}(a).$$

## 3 Induction Cannot Be the Complete Story

The naive story of next-instance induction does not yield an answer to the question about *how* scientists make predictions. Here is an argument for that conclusion: The method of induction says, roughly, that regularities that are consistent with the past data should be extrapolated to new instances. The problem is that many different regularities are consistent with any finite set of data, and these yield incompatible predictions when extrapolated to the next instance. So, the method of induction is, at best, an incomplete description of how scientists make predictions. Here is the argument in detail. Consider the data plotted in Figure 2: The straight line through the data points is the

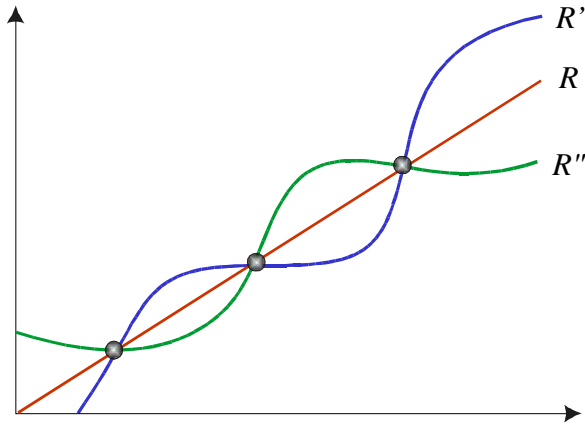


Figure 2: Three regularities fit the same data points

graphical representation of the regularity we earlier expressed by the equation  $dist(b) = 2 dist(a)$ . The other two curves represent regularities that have quite different equations (which I will not try to write down - there is no need). Each *hypothesis* describes a regularity. Thus Figure 2, shows that there are a number of logically incompatible *hypotheses* that conform to any data set. In fact, for any set of data points (which is always a finite set) there will be an infinite number of hypotheses that *fit* the data. This fact is the *underdetermination* of hypotheses from data. It raises a difficulty. Different hypotheses (out of those that fit the data) will yield different predictions. Thus, there is no *unique* prediction yielded by the method of next-instance induction. And so, it does not completely describe how predictions are actually made, since scientists do come up with single predictions.

#### 4 Prediction in Two Steps

We need to take a more serious look at what goes on in science. Let us return to the beam-balance example. A more realist picture of what goes on in science is that predictions are made in two steps. We start with a background *theory*. In our case we start with Newton's theory of motion, which tells us about forces and how they produce motion. In the case of a beam, Newton's theory tells us that it will remain mo-

tionless (i.e., balance) when the forces and the leverage of the forces exactly balance each other. Leverage is an idea familiar from other applications. For example, we all know how it is possible to "magnify" the effects of a force using a rigid body such as a crowbar. A long rigid beam may lever a large boulder if the point at which we push down is a long distance from the fulcrum compared with the distance of the fulcrum to the end of the beam applying a force to the boulder. By way of example, imagine that the downward force is applied at a point four times the distance from the fulcrum as the boulder (Figure 3). Then the upward force magnifies four times. Of course, I have to apply the downward force through a distance four times what the boulder actually moves, so the work done by me is equal to the work done on the boulder. This is required by the law of conservation of energy.

The same principles apply to our beam balance example. The forces applied to the beam are due to the gravitational forces of the two objects. If  $m(a)$  is the mass of  $a$ ,  $m(b)$  is the mass of  $b$ , and  $g$  is the gravitational field strength, then  $a$  exerts a gravitational force of  $m(a).g$  on the beam at the point at which it is hung, and  $b$  exerts a force of  $m(b).g$  at the point at which it is hung. Now focus on the object  $a$ . If the beam is to balance then the forces acting on  $a$  must balance. That is, the upward leverage of  $b$  on  $a$  must balance the downward gravitational force  $m(a).g$ . By the principles of leverage just dis-

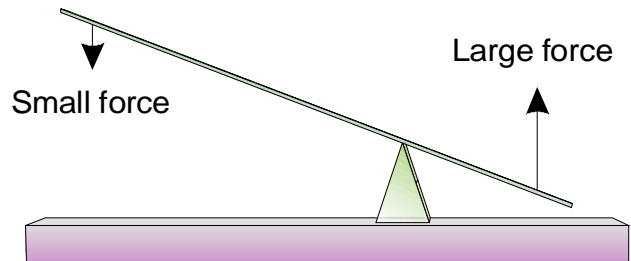


Figure 3: The principle of leverage is most commonly about the magnification of forces

cussed,  $b$  is exerting an upward force on  $a$  equal its downward force magnified by the ratio of the distance  $dist(b)$  to  $dist(a)$ . The background theory, viz. Newton's theory of mechanics, tells us that these two forces must be equal:

$$\frac{dist(b)}{dist(a)} m(b)g = m(a)g$$

If we multiply both sides of this equation by  $dist(a)$  and divide both sides by  $m(b)g$ , and simplify, we derive the equation:

$$dist(b) = \frac{m(a)}{m(b)} dist(a)$$

This completes the first step of the prediction task. Note that this derivation does not provide us with the value of the mass ratio. For that reason, this is only the *first* step toward making a prediction. What we have obtained is a whole *family* of hypotheses, which we will call a *model*. Each hypothesis is represented on the diagram in Figure 5 as a straight line that passes through the origin (the point at which both  $dist(a)$  and  $dist(b)$  are zero). Each of these lines has a specific slope which is equal to the mass ratio  $m(a)/m(b)$ . The quantity  $m(a)/m(b)$  is an adjustable *parameter*, and each possible numerical value is a *parameter value*. If we could measure this parameter, we would thereby pick out a single hypothesis from the family. Then we could make predictions using that particular hypothesis. That is where the background data come into the story.

The *second step* in the prediction process is to select a particular hypothesis from the model using the data. This second step resembles the process described by classical induction. The method measuring the parameters consists in *eliminating* the hypotheses from the model that do not fit the data, and for that reason it is often called *eliminative induction*. In our example, there is only one hypothesis in the model that fits the data exactly, which is the hypothesis for which the mass ratio is equal to 2. That is, the fitting of the model to the data yields the value 2

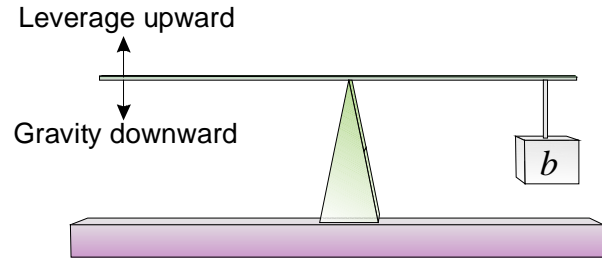


Figure 5: The beam balances because the leverage acting upward balances gravitational force downward

for the parameter  $m(a)/m(b)$ . It shows, in other words, that  $a$  is twice as heavy as  $b$ . (This is why the beam balance is commonly used for measuring mass). Now, we have the specific predictive hypothesis:

$$dist(b) = 2 dist(a).$$

It was first introduced in section 2 as providing the solution to the prediction task. We see that the route to the prediction is more circuitous than we first thought, but note that the known data is now playing an essential role. However, the reasoning by which we arrive at the prediction does involve a missing premise after all - namely, a statement that specifies the model. The inclusion of this missing premise has enabled us to answer the question about *how* predictions are made, but it leaves many unanswered questions about why this is the right prediction to make. Why that

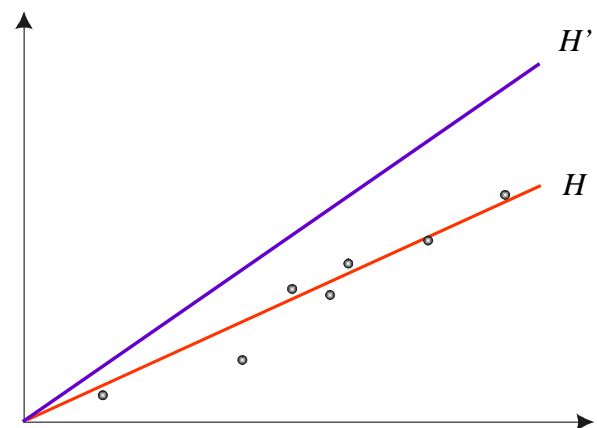


Figure 4: Neither hypothesis fits perfectly, by  $H$  fits better than  $H'$

model as opposed to another? Why that theory as opposed to another? These are deeper questions, for as I have pointed out, hypotheses, models, and theories, are all underdetermined by data - they transcend experience. Much of the philosophy of science is an attempt to move closer to an answer of these questions.

We must always be reminded that it may be a wasted effort to try to answer the why-question before we have adequately answered the how-question correctly. For that reason, there are two complications to the story of how predictions are made that we must deal with. The first is because theories do not uniquely determine the model; the second is that data is always error-ridden or noisy, so that none of the hypotheses in a model fit the data exactly. I will deal with these in that order.

## 5 One Theory, Many Models

In the beam balance example, there were many assumptions made to derive the model from Newton's theory of mechanics. For example, we ignored the leverage applied by the mass of the beam itself. This assumption would be justified if the beam were massless, or if it were of uniform mass and supported exactly in the center. As another example, we ignored the possibility that other forces, besides gravitational forces, were acting. We tacitly assumed that there were no electrostatic forces, no puffs of wind, no magnetic forces, and so on. A third example is the tacit assumption that the gravitational field strength,  $g$ , was the same on both sides of the beam. We know that  $g$  is different at different places on the surface of the earth (e.g., is greater at the poles than at the equator). For a small beam balance the two masses will be at approximately the same place, so  $g$  will be approximately the same, but not exactly the same. All such simplifying assumptions, when they are made explicit, are called *auxiliary assumptions*.

In practice, it is impossible to list all auxiliary assumptions, and often scientists do not make them explicit.

These auxiliary assumptions do not come solely from the theory. Newton's theory of mechanics does not tell us whether a beam has uniform density, or whether it has been properly centered. The theory does not tell us whether there are electromagnetic forces acting, or whether the gravitational force field is uniform, or whether air currents are having an effect. Nor do auxiliary assumptions come solely from the data. The data do not pick out a unique set of auxiliary assumptions from among the alternatives. For example, if  $a$  is subject to a gravitational field strength of  $g_1$  while  $b$  subjected to  $g_2$ , then the model derived from the theory is:

$$\text{dist}(b) = \frac{m(a)g_1}{m(b)g_2} \text{dist}(a).$$

This model defines exactly the same family of curves (all straight lines passing through the origin). The only difference will be a difference in interpretation: In this model, the slope of the straight line passing through all the data points (equal to 2) is interpreted as the value of the ratio of the weights rather than the ratio of the masses.<sup>1</sup> Other data (amongst the background data) may help to decide between auxiliary assumptions. For example, we could test the assumption that the gravitational field strength is uniform by seeing whether a single mass stretches a spring different amounts when it is moved from place to place. Yet this test will also introduce its own auxiliary assumptions. Eventually, we are going to find auxiliary assumptions that are not derivable from theory plus data. If this is correct, if there are auxiliary as-

<sup>1</sup>Weight is the *force of gravity*, equal to the mass times the gravitational field strength - the weight of an object is different on earth than it would be on the moon because the gravitational field strength is different, but its mass would not change.

sumptions that are not wholly derivable from theory plus prior data, then there will be many models that are equally compatible with the theory plus all prior data.

## 6 Dirty Data

The other complication of our previous picture is required by the fact that data are never so clean as to exactly fit any of our hypotheses. There is always some error due to the imprecision of our measuring instruments. It is not possible to read off the exact position at which a mass is suspended from a beam. For example, there is an error of parallax because the needle that marks the position of a mass will line up with different marks on the beam depending on the viewing angle. It is not possible to make the viewing angle exactly the same every time. And there is a limit to the precision to which we can read a scale anyway. It is impossible to tell the difference between 1.5000 cm and 1.5001 cm just by looking. There are more precise methods of measuring distance, but none are completely error free. Then there is error because our auxiliary assumptions are not *exactly* true. No matter how careful we are, our beam will never be suspended exactly in the middle. So, this must always produce *some* error present in the data.

Interestingly, this fact need not alter our picture of prediction very radically. All we need to change is the method, in the second step, of measuring the parameters of the model. Instead of choosing the hypothesis in the model that exactly fits the data (for in general, none will fit exactly), choose the hypothesis that *best* fits the data. In statistics, the method of choosing a best fitting hypothesis (e.g., the method of least squares) receives an exact quantitative formulation, but these details need not concern us here. For our purposes, it is enough to agree that some hypotheses have better goodness of fit with the data than do other hypotheses. For example,

given the data represented in Figure 5, evidently  $H$  fits the data better than  $H'$ . It is not a great leap to agree that  $H$ , or some hypothesis close to it, fits the data better than any other straight line passing through the origin. Once we have singled out a hypothesis in the model as the best fitting hypothesis, the second step of the prediction procedure is complete.

## 7 'Fuzzy' Prediction

In my first formulation of the problem of prediction, predictions were something *deduced from, entailed by, or demonstrated by*, a hypothesis.<sup>2</sup> When such a logical consequence is true, the prediction is successful. The existence of noisy data requires us to be more liberal about what counts as a successful prediction. For us, success now comes in degrees depending on how well a curve fits the unseen data. The unseen, or "predicted", data is not deduced from the curve in any sense since it does not lie on the curve. At best, the points on the curve are the ones predicted, and these are false. But false predictions may fit the facts extremely well.

Also, statistical or probabilistic hypotheses do not make any observable predictions in the deductivist's sense. For example, the familiar hypothesis that the coin I'm about to toss is a fair coin has no observational consequences. It "predicts" that the relative frequency, or proportion, or 'heads up' obtained in a large number of tosses is probably around 50%, but a statement of probability is not a statement of data. The logical consequences of probabilistic hypotheses are statements of probability, which are not observational. Statements of relative frequency are

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<sup>2</sup>Strictly speaking, we can only deduce the value of a  $Y$ -variable from a predictive hypothesis once we are given an  $X$ -value. This is analogous to the fact that we can only deduce that Fred is white from "All swans are white" once we are given that Fred is a swan.

observational, but a probabilistic hypothesis does not entail the truth of any such statements.

It is natural to count observational statements that are given high probability by a hypothesis as being predicted by the hypothesis to some degree. Statisticians call the probability of data given a hypothesis the *likelihood* of a hypothesis relative to the data in question. “Likelihood” is used here as a technical term, and should not be confused with the probability of the hypothesis given the data. Likelihood is a probability, but of the data, not the hypothesis. We are measuring the success of a hypothesis in predicting data by its likelihood relative to those data. In fact, likelihood is the most general way to measure the *fit* of a statistical hypothesis with a data set. In the limiting case in which the fit is perfect, the likelihood is one, while a likelihood of zero is the worst possible fit. Perfect prediction is perfect fit, but a high degree of fit is a praise-worthy achievement in a noisy world. Witness probabilistic theories like QED (quantum electrodynamics). We need an epistemology that recognizes the achievements of such theories.

## Summary

1. *The philosophical problem of prediction* is the problem of saying (a) *how* predictions are made, and (b) why predictions made in that way should be accurate.
2. The *naive approach* to the problem of prediction is to present next-instance induction as answer to the how-question: The regularities observed in the past, will continue to apply in the future.
3. The idea of induction - the idea of extrapolating past regularities to the next instance - fails because any finite set of data will *conform* to many regularities, each of which will recommend a different prediction. Without some criterion to choose from among the dif-

ferent regularities, the method of induction is, at best, incomplete.

4. *Terminology*: A *predictive hypothesis* describes a regularity that is specific enough to be used for prediction. A *model* is a family of hypotheses. A *theory* states very general principles, or laws (such as the principle of leverage), and is even less specific than a model (it may be usefully thought of as a family of models).
5. A two-step procedure provides a better picture of how predictions are made. *Step 1*: Use a background theory to derive a model (= family of hypotheses). *Step 2*: Use the data to select the hypothesis from the model that best fits the past data.
6. This simple two-step picture must be complicated in two ways. The first complication arises from the fact that there will be many models that are compatible with the background theory plus prior data. The second complication is due the noisy nature of real data. Observational errors, and other errors, make it unlikely that any hypothesis in a model will fit the data exactly. Thus, Step 2 only involves selecting the *best* fitting hypothesis in the model, without assuming that the best fitting hypothesis fits the data exactly.
7. The problem of prediction is a problem about how predictions are made and why they are successful. But the notion of successful prediction requires only good fit with unseen data, not perfect fit. A prediction, in this more liberal sense, need not be a logical consequence of the hypothesis. This allows us to extend the problem of prediction to encompass statistical, or probabilistic, theories.