Econophysics: A simple explanation of two-phase behaviour

Financial markets exhibit two phases of behavior, an equilibrium phase, and an out-of-equilibrium phase (ref 1). Such behaviour can arise from a sequence of unrelated events that each have a volatility described by a long-tailed bell-distribution.

The details of the financial market analysis in question (ref 1) are equivalent to the following. Consider any two independent identically distributed quantities, x_1 and x_2, and define the variable of interest to be $\Omega \equiv x_1 + x_2$. Also define

$$\Sigma \equiv \tfrac{1}{2}\,|x_1 - \langle x\rangle| + \tfrac{1}{2}\,|x_2 - \langle x\rangle|,$$

where $\langle x\rangle \equiv \tfrac{1}{2}x_1 + \tfrac{1}{2}x_2$. Σ is called the local noise intensity. Now calculate (Σ, Ω) for many pairs of (x_1, x_2), and plot the points (Σ, Ω) on a scatter diagram. The phenomenon to be explained is that for small values Σ, the Ω-values are clustered near 0, while for high values of Σ, the Ω-values are clustered near $+c$ or $-c$, for some $c > 0$.

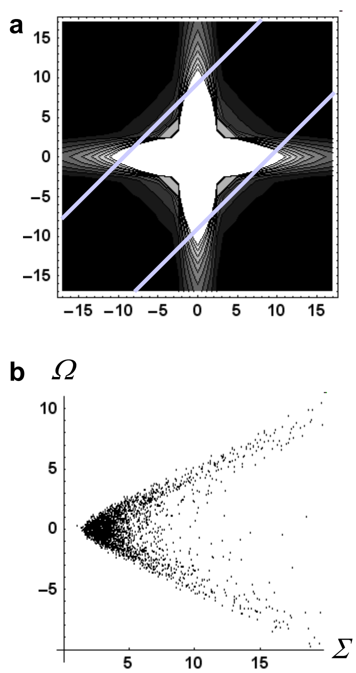

Figure 1 The existence of two distinct phases can be a direct consequence of long-tailed distributions of independent random variables (ref 5, 6). **a**, A contour plot of the joint distribution of two independent Lorentzian random variables. Since the expression for Σ reduces to $\tfrac{1}{2}|x_1 - x_2|$, the condition $\Sigma = \tfrac{1}{2}c$ is equivalent to $|x_1 - x_2| = c$. This gives the equations of the two sloping lines. At the points of intersection with the ridges, the quantity $\Omega = x_1 + x_2$ has values clustered in two peaks near $+c$ and $-c$. For values of Σ close to 0, Ω has values clustered around 0 in a single peak. **b**, When the definitions of Σ and Ω are generalized to N random variables, the effect is still present. The plot shown is for $N = 100$. Each point is determined by a randomly generated array of 100 numbers. There are 5,000 points in total, where most are clustered at the left (the so-called equilibrium phase). The continuation of the effect for larger N is expected from the fact that the Lorentzian is stable, which means that any sum of Lorentzian variables is Lorentzian (ref 3, 4).

We found that such phenomena can be explained by long-tailed bell-distributions, such as the Lorentzian (Cauchy). For two Lorentzian variables, x_1 and x_2, the joint distribution $P(x_1)P(x_2)$ is represented in Fig. 1a as a contour plot. The contours at the centre of the plot (not shown) are concentric circles, and these circles would continue outwards if the distribution were Gaussian. But the striking feature of the Lorentzian distribution is the presence of the four ridges. Benoit Mandelbrot in 1963 (ref. 2, Fig. 4) identified this feature as general characteristic of stable non-Gaussian distributions (ref. 3, 4).

The two-phase behaviour of the conditional distribution $P(\Omega|\Sigma)$ follows by a simple argument in the special case in which Ω and Σ are defined in terms of two variables (see the caption of Fig. 1a). But does the explanation generalize to N variables? We used computer simulations to answer this question for a variety of values of N and a number of different probability distributions. When the variables $x_1, x_2, \ldots, x_N$ are independent and Lorentzian, the two-phase phenomena showed up in every case. The results for $N = 100$ are shown in Fig. 1b.

Long-tailed bell-distributions are frequently used to describe the volatility of financial market prices (ref 2, 5, 6). We show that the same statistical model produces the two-phase effect. The recent discovery of the effect (ref. 1) is not for prices, but for the number of seller-initiated or buyer-initiated trades. It is therefore unknown at this time whether our explanation applies in this case.

Our lesson is this. Suppose x_1 is the price of cotton in the U. S. and x_2 the price of tea in China. Such a composite system would exhibit two-phase behavior if the probabilities were Lorentzian, and the effect would be an artifact of the distributions. In other cases, a deeper explanation may exist; however, the mere presence of the effect does not provide evidence of this.

MALCOLM FORSTER, BRAD HALFPAP
Department of Philosophy and Wisconsin Center for Education Research, University of Wisconsin, Madison, Wisconsin 53706, USA

e-mail: mforster@wisc.edu, halfpap@wisc.edu

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